UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2020

MATHEMATICS

PAPER-V (New Syllabus)

Time Allotted: 1 Hour | Full Marks: 25

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer Question No. 1 and any *one* from the rest

1. (a) Find the maximum value of the function $f(x) = x^2e^{-x}$, $x > 0$. 1

   (b) Show that $\lim\limits_{x\to 0} f'(x)$ does not exist, where $f(x) = \begin{cases} x^2\sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. 1

   (c) Let $f, g : D \to \mathbb{R}$, where $D \subset \mathbb{R}$. Let p be an accumulation point of D. If $\lim\limits_{x\to p}$ exists finitely and be equal to zero and g be bounded on D, then prove that $\lim\limits_{x\to p} f(x)\,g(x) = 0$. 1

2. (a) If $f : \mathbb{R} \to \mathbb{R}$ satisfies $|f(x) - f(y)| \leq |x - y|^2$ for all $x, y \in \mathbb{R}$, prove that f is a constant function. 3

   (b) If $f(x)$ is a continuous real valued function on an interval I, then prove that the set 4

   $$f(I) = \{f(x) : x \in I\}$$

   is an interval or a singleton set.

3. (a) If $y = \sin(m\sin^{-1}x)$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. 3

   (b) State and prove Sandwich theorem. 4

4. (a) Show that the series 3

   $$\sum \frac{3.\,6.\,9.\,\cdots.\,3n}{7.\,10.\,12.\,\cdots\,.\,(3n+4)}\; x^n, \quad x > 0$$

   converges for $x \leq 1$, and diverges for $x > 1$.

   (b) If a function $f : [a, b] \to \mathbb{R}$ be injective on $[a, b]$ and f has the intermediate value property on $[a, b]$, then f is strictly monotone on $[a, b]$. 4

GROUP-B

Answer any *one* question

5\. (a) For the function $f(x,y)=\begin{cases} xy & \text{if } |y|\le |x| \\ -xy & \text{if } |y|>|x| \end{cases}$, $2\frac{1}{2}$

prove that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

(b) Let u be a function of x and y satisfying $x=\theta\cos\alpha-\phi\sin\alpha,\ y=\theta\sin\alpha+\phi\cos\alpha$ ($\alpha=$ constant). Prove that $2\frac{1}{2}$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial \phi^2}$$

6\. (a) If $\alpha,\ \beta,\ \gamma$ be the roots of the equation in λ, such that $2\frac{1}{2}$

$$\frac{u}{a+\lambda}+\frac{v}{b+\lambda}+\frac{w}{c+\lambda}=1,$$

then find the value of $\dfrac{\partial(u,\ v,\ w)}{\partial(\alpha,\ \beta,\ \gamma)}$.

(b) State and prove Schwarz's theorem. $2\frac{1}{2}$

GROUP-C

Answer Question No. 7 and any *one* from the rest

7\. (a) Examine the curve $(x^2+y^2)x-2y^2=0$ for singular points at the origin. 1

(b) Find the rectilinear asymptotes to the curve $y=xe^{1/x^2}$. 1

(c) Find the radius of curvature of $y=4\sin x-\sin 2x$ at $(\pi/2,\ 4)$. 1

8\. (a) Find the pedal equation of the curve whose parametric equations are given by 3

$$x=a(3\cos\theta-\cos^3\theta)$$
$$y=a(3\sin\theta-\sin^3\theta)$$

(b) Find the asymptotes of the curve 4

$$y=(a-x)\tan\frac{\pi x}{2a},\quad a\in\mathbb{R}\setminus\{0\}.$$

9\. (a) Find the polar subtangent of the equiangular spiral $r=ae^{\theta\cot\alpha}$. 3

(b) The loop of the curve $2ay^2=x(x-a)^2$ revolves about the line $y=a$. Using Pappus theorem, find the volume of the solid generated. 4

10\.(a) Find the area included between the $x^2+2y^2=4$ and $2x^2+y^2=4$. 4

(b) Show that the cardiodes $r=a(1+\cos\theta)$ and $r=a(1-\cos\theta)$ cut orthogonally. 3

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